SETTING CAPITATED RATES FOR CHILD WELFARE PROGRAMS

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December 1998

Abstract

Child welfare services currently operate in an environment characterized by increasing need for services, and calls for cost containment and system reform. To survive in this environment, foster care agencies will have to reexamine their use of resources to ensure that they are used to achieve the best possible outcomes for the children in their care.

The authors suggest that capitated payments, most often used in health care delivery, are a potentially useful approach for foster care agencies. Unlike the current fee-for-service system, capitated payments are obtained prospectively, permitting the agency to engage in long-range planning for the allocation of resources, and to purchase alternative services as well as foster care. This paper discusses issues related to setting a capitation rate and how to estimate the maximum rate of expenditure permissible for solvency under the proposed capitation rate.

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ISSN: 1097–3125

Setting Capitated Rates for Child Welfare Programs

Fred H. Wulczyn and Eileen Sheu

Introduction

Not since 1980, when Congress passed the Adoption Assistance and Child Welfare Act of 1980, has so much attention been paid to how the government finances child welfare services. The impetus for this renewed interest has several sources. First, and perhaps foremost, the federal government has enacted block grants for a number of important programs including AFDC (now TANF) and has actively considered doing so for child welfare services. Inasmuch as foster care, the largest federal child welfare program, is financed through per diem rate structures in an entitlement context, block grants would force a fundamental shift in the way states disburse child welfare funds. Second, providers of child welfare services, including not-for-profit agencies and units of local government, have grown increasingly frustrated with the rigidly categorical nature of funding. This frustration is best typified by the "funding dilemma" embedded in the way foster care is currently financed: because any provider that lowers foster care utilization below some historical baseline captures fewer per diem payments, programmatic success results in declining revenues.

The current emphasis on fiscal reform, coupled with clamors for broad system reform, has created an environment in which the basis for funding child welfare programs has been called into question. Yet, the need for child welfare services continues to increase. In order to accommodate these tensions, foster care providers (including not-for-profit contractors as well as public child welfare agencies) cannot simply proceed on the basis of old assumptions, but must carefully reexamine their utilization of resources to ensure that the best possible outcomes are achieved for the children they serve.

Changing the child welfare system depends to some degree on the availability of alternatives to the per diem payment system. The use of capitated or prospective funding is one solution to the problem of improving outcomes within the increasingly common context of fiscal constraint. Capitated payments involve a fixed or limited reimbursement for the cost of providing services in exchange for flexibility in service provision, typically bundled together with care management strategies that are used to govern service delivery and utilization with the expectation that expenditure patterns will change because of the increased attention to the process of care.[1] Although the capitated payment exposes the foster care provider to a degree of financial risk, the advantage over the current fee-for-service system is that funds are obtained prospectively based on estimates of service utilization over a fixed period of time. Thus, agencies can engage in long-range planning to

[1] *Managed care* is the term most frequently associated with strategies that combine fiscal and programmatic reforms. In a study of managed care programs, Wulczyn and Orlebeke use a dual-axis continuum to isolate the particular combination of fiscal and programmatic reform involved in child welfare reforms. Wulczyn, F. and Orlebeke, B., "Managed Care and Child Welfare Reform: Assessing Progress and Developing Evaluation Framework," Chicago: Chapin Hall Center for Children, 1997.

allocate resources more effectively. This relative autonomy provides agencies with the opportunity to either purchase foster care or divert funds to community-based alternative services for which reimbursement is currently more difficult to obtain.

If and when child welfare systems move in the direction of greater flexibility with respect to funding and greater accountability with respect to program outcomes, it will be absolutely essential that fiscal and program managers have a solid understanding of the actuarial theory that goes into the development of a prospective payment system.

With regard to foster care, the challenge for the child welfare agencies is to channel resources towards those services that yield the best outcomes for children and their families without spending above the capped amount. Doing so requires a firm understanding of how capped rates are established and how resource utilization relates to the outcomes accomplished. In turn, this understanding is dependent on a clear definition of terms and a dynamical model that explicitly connects funding levels to service outcomes.

The purpose of this paper is to address technical issues associated with developing a capitated or prospective payment system in which the total amount of revenue is established prior to the actual delivery of services. Although they are widely used in the health care system, there is little or no practical experience with these rate-setting models in the child welfare field. With this in mind, this paper will focus on three essential issues: (1) how to estimate foster care utilization; (2) how to set a capitation rate; and (3) how to calculate the maximum rate of expenditure permissible to maintain solvency under the proposed capitation rate. The discussion is necessarily technical. However, if and when child welfare systems move in the direction of greater flexibility with respect to funding and greater accountability with respect to program outcomes, it will be absolutely essential that fiscal and program managers have a solid understanding of the actuarial theory that goes into the development of a prospective payment system. We begin this exposition with a definition of a capitated payment and the parameters that determine over time the total revenue that flows through the foster care system.

Defining the Capitation Rate

A *capitated payment* is a preset reimbursement for services over a specified length of time; it must be comparable to the actual cost of providing the services.[2] The total revenue to the child welfare agency (or cost of providing child welfare services) can be represented as an aggregate of three variables—volume, duration, and unit cost. Volume refers to the number of children requiring services (admissions); duration refers to length of stay in foster care (or number of care days used per child); and unit cost refers to the average cost per day of providing services to a child.

$$\text{Total revenue} = f\,(\text{Volume, Duration, Unit Cost})$$

These parameters are the same three parameters that determine total costs in fee-for-service systems. The difference between the two fiscal systems has do to with which parameter is "fixed" prospectively. In the per diem system found in most foster care programs, the unit cost is the only parameter that is fixed ahead of time. Volume (number of admissions) and length of stay (duration) vary over the course of a fiscal year (and beyond), so that the total revenue is not known until the end of the fiscal year. In a prospective or capitated payment system, total revenue is determined at the beginning of the fiscal year. In order to establish revenue amounts prospectively, information about volume, duration, and unit costs must be projected. These projections require

[2] The term *capitated payment* needs some greater clarity. Technically, capitated payment is a term used to describe reimbursements that cover a population at large where the range of potential service utilization ranges includes individuals who are not expected to use services at all. Our use of the term in this context is generic in the sense that we are referring to any number of fixed or prospective reimbursement methodologies.

knowledge about historical utilization and cost patterns. Moreover, by fixing the revenue on the left-hand side of the equation, capitated payments require greater control over all three revenue parameters.

Establishing Baseline Utilization

The first step in establishing a prospective payment involves making estimates of the three parameters that determine revenue. The rate-setting model we present below stresses the duration parameter. That is, we focus our discussion on estimating rates that are driven by length of time that children who are already in care at the start of a fiscal period remain in care. We have taken this approach for three reasons. The first concerns the length of time that children remain in foster care. Given the current dynamics, the bulk of foster care expenditures tend to be devoted to children already in care. Thus, a methodology focusing on duration is particularly relevant to an important subset of the foster care population. Second, notwithstanding the Adoption and Safe Families Act of 1997, the children in care at any point in time represent a programmatically unique population that typically receives less attention than the children at risk of entering foster care. Finally, the focus on the children in care makes certain simplifying assumptions possible, and thereby makes the exposition of the model more tractable. Among these simplifying assumptions, the fact that the first day of care is by definition the same for all covered children in care is particularly important. Admissions to foster care can, of course, happen at any time during a year. This means that in a given fiscal year, two children with the same projected length of stay can have substantially different effects on the revenue projections for that year. In the end, the methods for handling the duration and volume parameters are very similar.[3] However, we have opted for an approach that amplifies the basic ideas.

Reductions in the average length of stay in foster care imply success in achieving permanency for the child, either through adoption or reunification with the biological family after the tensions leading to abuse or neglect are defused. Because prospective payments permit the purchase of such family reunification programs, as well as preventive or other community-based services, and utilization of these services is expected to result in an acceleration of the rate of discharge of children from foster care, overall expenditures for foster care will be reduced. The net outcome is, in effect, a redistribution of funds that benefits the child, the family, and the community, provided that total revenue is unaffected by the fact that foster care utilization falls below the baseline.

Because the actual number of children receiving foster care each day directly influences the rate of resource utilization, an accurate description of the discharge rate for a population of children in care is necessary. This description can range from the simple (one child leaves foster care every five days) to the complex (a Kaplan-Meier survival curve calculated from 10,000 cases throughout an entire state). In either case, the basic procedure involves modelling the rate of discharge for a stock of children. By using an agency's historical data, a graph of the number of children remaining in care versus the number of days elapsed can be constructed and used to project expenditures for a fixed period. In our example, shown in Figure 1, a steady discharge, dependent upon the number of children in foster care, is assumed.[4]

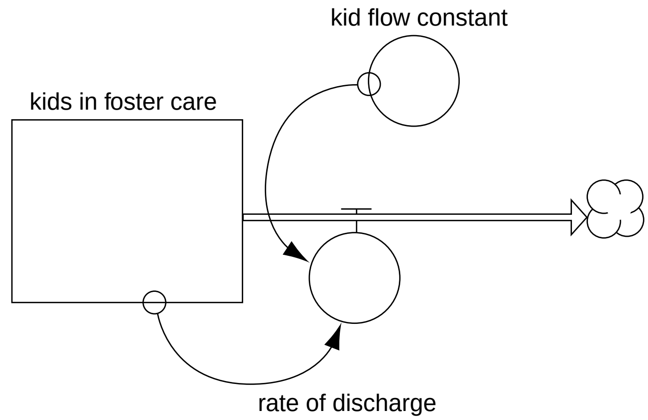
The primary indicator for how fast children are being discharged from foster care is k, the kid flow constant, which represents a constant of proportionality between the instantaneous rate of change in the population of the initial stock of foster children with respect to time and the number of children left in foster care at that time. Knowledge of the magnitude of this constant allows one to

[3] In point of fact, the fiscal tension produced by the capitated environment due to limited revenue is expected to ensure a degree of prudence in the regulation of volume (placement prevention), duration (expedited discharge), and unit costs (stepping down).

[4] The diagrams used throughout this section are intended to depict the stock and flows that underpin the rate-setting task. In Figure 1, the rectangular box depicts the stock of children in care at the beginning of the rate period. The flow is the number of children leaving the stock per unit of time, while the kid flow constant is the factor that determines the actual magnitude of the flow.

F I G U R E 1

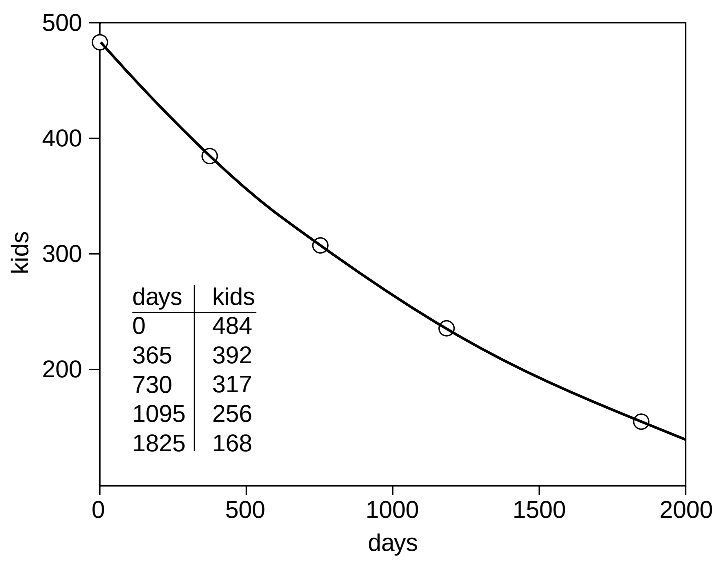
Representation of the Steady Discharge of Children



Stella II diagram representing the steady discharge of kids. The rate of discharge is dependent on the number of children from the initial stock remaining in foster care at a given time.

F I G U R E 2

Rate of Discharge



make predictions about the number of children in foster care on any given day (see Appendix, equation 6) over the rate period, assuming that future usage will be comparable to prior utilization.

A number of children are identified from agency data as being in care on the predetermined day representing the start of the rate period, defined to be t=0. These children represent the initial stock of children whose rate of discharge will be examined. As time progresses, the number of these same children remaining in foster care after designated intervals of time (e.g., t = 180 days, t = 365 days, t = 720 days, and t = 1,095 days) are determined, and a graph of the number of children in care vs. time elapsed is constructed in preparation for extrapolation of the rate constant. (See Figure 2.)

Because of the difficulty of extracting a constant of proportionality from a function that curves (as is the case in Figure 2), the data is mathematically transformed to an easier-to-manipulate linear form by taking the natural logarithm of the number of children in care and replotted. A graph of the transformed data—with days plotted on the horizontal axis, and the natural logarithm of the number of foster children plotted on the vertical axis—is constructed, and the best-fitting straight line through all of the points determined (Figure 3 on page 5). The slope of this line is the magnitude of *k*, having units of days.[5]

After graphing, two points lying as far apart as possible on the line are selected and denoted by ($days_a$, $kids_a$) and ($days_b$,$kids_b$). The slope of the line is then determined as follows:[6]

$$\frac{d[\text{kids}]}{dt} = -k[\text{kids}]$$

Estimating the Rate of Expenditure

As mentioned in the previous section, expenditure over time is directly related to the number of children experiencing placement

[5] It ought to be noted that k, a constant, has the same value whether it is extracted from the curve or from the linearized transform.

[6] Note that enclosure of a quantity in vertical bars means that the absolute value of the quantity is imposed—i.e., that the result of the calculation inside the vertical bars is changed to a positive number if negative; otherwise, it remains positive.

in foster care each day. This is illustrated in Figure 4 on page 5.

Using Equation 18, derived in the Appendix, the yearly expenditure can be directly estimated using this model:[7]

$$[\$] = [\$]_o + [pdr][kids]_o\left(\frac{1}{k}\right) - [pdr][kids]_o\left(\frac{1}{k}\right)e^{-k(t)}$$

Substitution of a value for “t” into this equation generates the total expenditure during the time period encompassing the first day through “t” days. Therefore, the total expenditure at the end of the first year, at the end of the second year (which includes the first-year expenditure), and at the end of the third year (which includes both the first-year and second-year expenditures) are determined and are subtracted from one another to obtain the projected expenditures for each year. (See Table 1.)

The sum of the expenditures over the five years must be comparable to the total revenue expected for providing foster care services; otherwise, appropriate adjustments in spending must be made. An estimation of the total funds available can be constructed from agency data by multiplying the per diem rate, the number of children served, and the average length of stay within the five-year period covered by the managed care plan.[8] For example, if an agency expects to serve 484 children in foster care over five years under a managed care plan, and the average length of stay is expected to be 1,149 days with a per diem rate of \$38.44 per day, the total revenue expected is estimated to be the following:

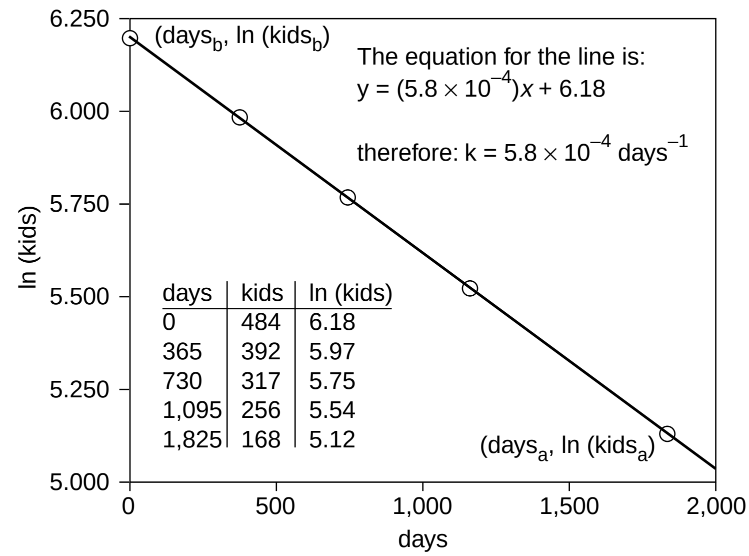
Total revenue = (484 children) × (1,149 days) × (\$38.44 per child per day) = \$ 21,377,099.04 for the five-year period

Setting the Capitation Rate by Frontloading Expenditures

The system of financing services described here is particularly amenable to such objec-

FIGURE 3

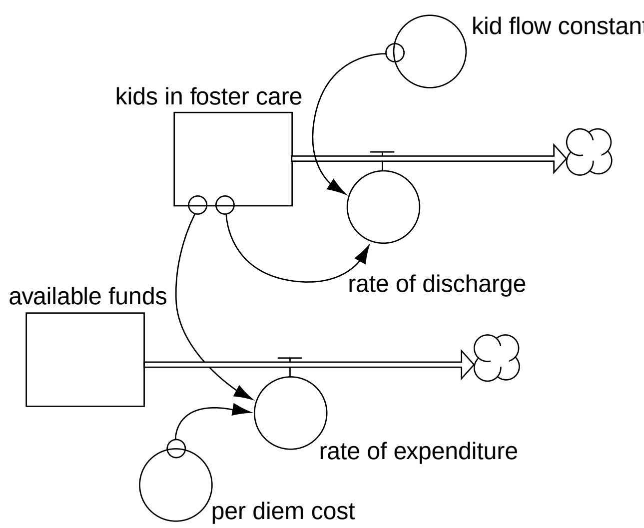
Determination of the Discharge Rate Constant



Plot of days in foster care vs ln (kids). The numbers used to construct this graph, as well as definitions relevant to calculation of the slope of this line, are included.

FIGURE 4

Relationship Between Utilization of Expenditures and Discharge from Foster Care



Stella II representation of the relationship between the utilization of expenditures and the discharge of children from foster care. The rate of expenditure is highly dependent upon the number of children in foster care.

[7] In this equation and all subsequent equations, \$ denotes expenditures and pdr refers to the per diem rate.

[8] The average length of stay is calculated from historical agency data. A number of children who are in care on a certain day representing the start of the rate period are selected and their continuous placement is tracked over a five-year period. A day representing the end of the rate period is selected and the average length of stay of these children within that window of time is determined. If a case is still open at the end of the rate period, the number of caredays is truncated on the close of the rate period.

TABLE 1

Projecting Yearly Expenditures

Using the equation:

$$[\$]_t = [\$]_o + [pdr][kids]_o\left(\frac{1}{k}\right) - [pdr][kids]_o\left(\frac{1}{k}\right)e^{-k(t)}$$

Substitute in:

t = 365	This results in $[\$]_1$, the total revenue over the first year
t = 730	This results in $[\$]_2$, the total revenue over two years
t = 1095	This results in $[\$]_3$, the total revenue over three years
t = 1460	This results in $[\$]_4$, the total revenue over four years
t = 1825	This results in $[\$]_5$, the total revenue over five years

To obtain :

The first-year revenue:	Use the quantity ($[\$]_1$).
Second-year revenue:	Evaluate the quantity ($[\$]_2 – [\$]_1$)
Third-year revemue:	Evaluate the quantity ($[\$]_3 – [\$]_2$)
Fourth-year revenue:	Evaluate the quantity ($[\$]_4 – [\$]_3$)
Fifth-year revenue:	Evaluate the quantity ($[\$]_5 – [\$]_4$)

See Appendix A for derivation.

tives as using less-restrictive services than foster care to meet the needs of children and families. Thus far, we have focused on accurately describing expenditures over the course of five years. Once the rate of utilization has been determined, a capitation rate can be established that will be sufficient to cover costs. Although the obvious method of capitating costs is to set payment at slightly less than the projected expenditure for each of the five years, this method is not likely to be as successful as when a desirable policy framework with clearly stated objectives, such as reducing average length of stay in foster care through use of family preservation programs, is incorporated into the capitation scheme. The procedure illustrated in this paper, therefore, will specifically incorporate the objective of reducing average length of stay in foster care by frontloading funds, so as to purchase alternative programs early in the rate period; it will limit the money available for foster care later in the rate period, providing concrete incentives for achieving reductions in average length of stay.

This scheme for reallocation of expenditures requires the prediction of monetary consumption in the absence of a managed care plan, which was illustrated in the previous section, as well as the prediction of monetary consumption under managed care over the same period of time. (See Figure 5.) An achievable reduction of the number of foster care days (estimated to be roughly 10 percent) is targeted, then a new baseline of expenditure utilization is projected for the proposed managed care plan. The savings expected to result from this reduction in foster care days are then frontloaded and used to purchase the additional services that will accelerate the rate of permanent placement for the foster children.

Earlier in this paper, we determined total cost for five years by multiplying the number of children to be served by the per diem cost and the average length of stay in foster care. To estimate the maximum amount of money available for foster care under the managed care plan, this total cost is now multiplied by 0.90, reflecting the 10 percent decrease in average length of stay:

Maximum revenue available for foster care for five years = {(number of children) × (pdr) × [(0.90)(ave. length of stay)]}

Because of the direct relationship between utilization of resources and number of children per day in the foster care system, shown earlier, an equation can be derived that predicts the total number of children that must be discharged in order to realize this new monetary goal. Therefore, the number of children to be discharged can be determined over the five-year period using Equation 27 from the Appendix:

$$[kids]_{proj} = \frac{k}{[pdr]}\left([\$]_{proj} - [\$]_o\right) - [kids]_o$$

where $[kids]_o$ = initial number of children; $[kids]_{proj}$ = number of children in foster care at time t; [pdr] = average cost per day; $[\$]_o$= expenditures on the first day ($[kids]_o \times$ [pdr]); and $[\$]_{proj}$ = maximum amount of money available under managed care and substituting t = 1,825 (for the five-year period) into the equation.

Because this equation is derived using the initial equation to determine the rate of discharge of children (Equation 9 of the Appendix) the same restrictions hold, that this equation is only valid when the maximum number of kids discharged is less than or equal to the initial number of children.

After the number of children to be discharged from foster care is determined, a new kid flow constant or rate of discharge, *k*, can be determined as before by calculating the slope between the points (t_o, $\ln[\text{kids}]_o$) and (t, $\ln[\text{kids}]_t$):

$$k = \left| \frac{\ln[\text{kids}]_o - \ln[\text{kids}]_t}{t_o - t} \right|$$

Using this new kid flow constant, a new baseline of expenditure can be projected for each year of the managed care program, as previously described in the section on estimating the rate of expenditure.

The capitated rate is determined by adding a percentage of the projected five-year savings to the estimated yearly expenditures, resulting in payments of approximately 43 percent of the total savings during the first year, 24 percent of the total savings the second year, 16 percent the third year, 11 percent the fourth year, and 6 percent the last year of the five-year period. The specific details are summarized in Table 2, and the results of an actual analysis for a hypothetical agency are tabulated in Table 3.

Feedback

Hypothetically, when the amount of money available is fixed, the problem of funding services focuses on regulating the balance between outcomes and expenditures over the duration of the contract. As an agency increases its ability to accurately monitor resource utilization relative to outcomes, the risk of instability is diminished.

An agency's position with respect to the balance between outcomes and expenditures can be determined by application of Equations 19 and 21, derived in the Appendix (note that boundary conditions or limitations upon the general applicability of these equations exist). The pertinent equations are listed below:

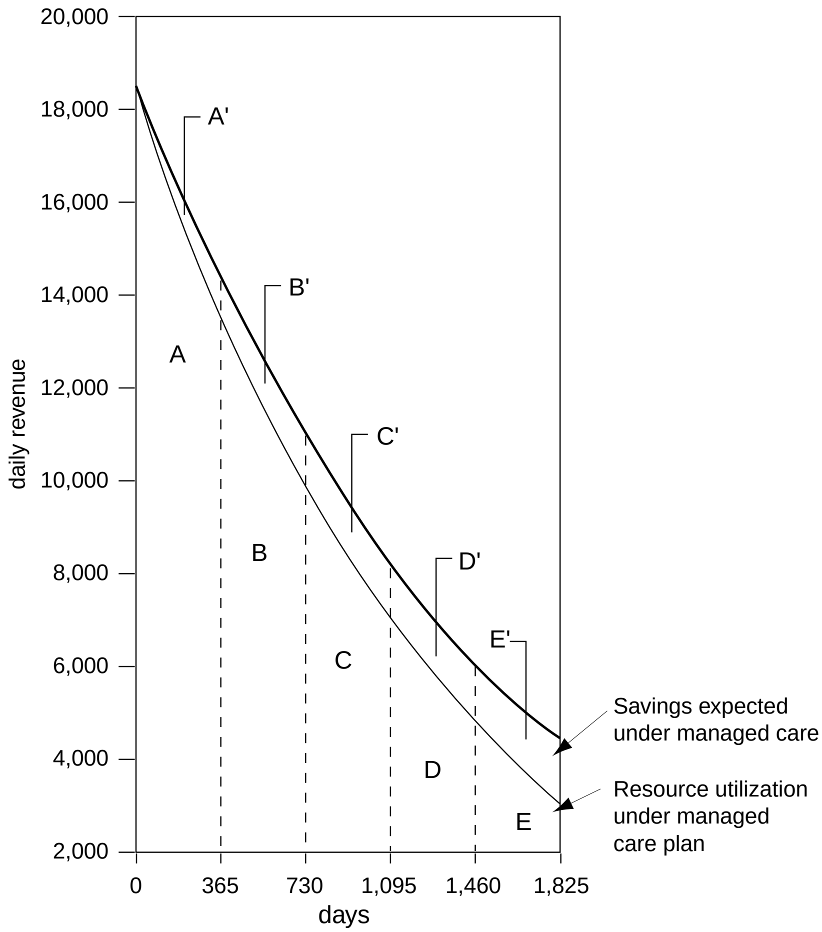
Use of the following two equations:

(a)

$$[\$]_t = [\$]_o + [\text{pdr}][\text{kids}]_o\left(\frac{1}{k}\right) - [\text{pdr}][\text{kids}]_o\left(\frac{1}{k}\right)e^{-k(t)}$$

F I G U R E 5

Resource Utilization With and Without a Managed Care Plan



Representation of the baseline of resource utilization expected in absence of the managed care plan (area between the bold, downward sloping line and coordinate axes); the projected baseline of resource utilization under managed care (area between the non-highlighted, downward sloping solid line and coordinate axes); and the expected savings (difference between two areas).

(b) $$[\text{kids}]_t = [\text{kids}]_o\, e^{-kt}$$

results in the expression:

$$[\text{kids}]_t = k\left(\frac{[\$]_t - [\$]_o}{[\text{pdr}]}\right) - [\text{kids}]_o$$

To determine the number of children to be discharged from foster care by the end of five years in order to meet the calculated financial goals, the following equation can be used:

$$[\text{kids}]_{proj} = k\left(\frac{[\$]_{proj} - [\$]_o}{[\text{pdr}]}\right) - [\text{kids}]_o$$

TABLE 2

Setting the Capitation Rate

First define:

A + A' = expected total consumption for first year of the baseline
B + B' = expected total consumption for second year of the baseline
C + C' = expected total consumption for third year of the baseline
D + D' = expected total consumption for fouth year of the baseline
E + E' = expected total consumption for fifth year of the baseline

A = projected year 1 consumption for foster care under managed care
B = projected year 2 consumption for foster care under managed care
C = projected year 3 consumption for foster care under managed care
D = projected year 4 consumption for foster care under managed care
E = projected year 5 consumption for foster care under managed care

A' = projected savings during first year under managed care
B' = projected savings during second year under managed care
C' = projected savings during third year under managed care
D' = projected savings during fourth year under managed care
E' = projected savings during fifth year under managed care

Scheme for reallocation of funds:

Capitated rate (revenue) for first year: A + A' + 0.6B' + 0.5C' + 0.4D' + 0.2E'
Capitated rate (revenue) for second year: B + 0.4B' + 0.3C' + 0.2D' +0.2E'
Capitated rate (revenue) for third year: C + 0.2C' +0.2D' + 0.2E'
Capitated rate (revenue) for fourth year: D + 0.2D' + 0.2E'
Capitated rate (revenue) for fifth year: E + 0.2E'

Having, after a period of time t, greater expenditures or more children than projected implies that an examination of factors contributing to overutilization of resources ought to be conducted. Procedural changes may also be required. On the other hand, accurate projection of expenditures or number of children bespeaks success in achieving the monetary and policy objectives of the managed care plan.

TABLE 3

Results of a Rate-Setting Analysis for a Hypothethical Agency

Year	Base Expenditures	Proposed Expenditures Under Managed Care	Savings	Redistributed Savings	Total Capitated Rate
1st year	$ 6,089,040.35	$ 5,982,434.97	$ 106,605.38	$ 667,430.84	$ 6,649,865.81
2nd year	4,927,424.69	4,665,973.87	261,450.82	362,816.57	5,028,790.44
3rd year	3,953,484.03	3,608,015.35	345,468.68	223,689.38	3,831,704.73
4th year	3,181,720.92	2,798,595.97	383,124.95	154,595.64	2,953,191.61
5th year	2,560,614.38	2,170,761.13	389,853.25	77,970.65	2,248,731.78
Total	20,712,284.37	19,225,781.29	1,486,503.08	1,486,503.08	20,712,284.37

Appendix

An equation describing the number of children remaining in foster care after an arbitrary number of days, assuming a constant flow of children, is derived in the following section. This method is based on the Stella II model, illustrated in Figure 1 of the main text.

First define:

t	number of days after the start of foster-care utilization study
$[\text{kids}]_t$	represents the number of children within the placement agency after t days
$[\text{kids}]_o$	represents the initial number of children placed with the agency at t=0.
k	kid flow rate constant. A larger k denotes a faster discharge of foster children from the program; k is determined by examining prior agency data
C	represents a constant of integration, the value of which can be determined by substituting in any condition where all values for the variables can be determined.

Therefore:

$$\frac{d[\text{kids}]}{dt} = -k[\text{kids}] \quad (1)$$

$$\frac{d[\text{kids}]}{[\text{kids}]} = -k\,dt \quad (2)$$

$$\int \frac{d[\text{kids}]}{[\text{kids}]} = -k\int dt \quad (3)$$

$$\ln[\text{kids}] = -kt + C \quad (4)$$

$$e^{\ln[\text{kids}]} = e^{-kt+C} \quad (5)$$

$$[\text{kids}] = e^{-kt}e^{C} \quad (6)$$

At t=0, representing the first day:

$$[\text{kids}]_o = e^{C}e^{-k(0)} \quad (7)$$

$$[\text{kids}]_o = e^{C}(1) \quad (8)$$

The kid flow equation becomes the following:

$$[\text{kids}]_t = [\text{kids}]_o\, e^{-kt}$$
$$\text{valid only for the interval } 0 \le [\text{kids}]_t \le [\text{kids}]_o \quad (9)$$

(Note that limitations or *boundary conditions* exist for this equation. Because you cannot have a negative number of children in foster care, solutions of less than zero for $[\text{kids}]_t$ are invalid and must be assumed to be zero. Correspondingly, solutions greater than $[\text{kids}]_o$ for $[\text{kids}]_t$ are also invalid because they result from negative values of time— nice in theory, but not representative of reality.)

The following is the derivation of an equation that describes the utilization of money by an agency with the previously defined flow of children. A Stella II model, illustrated in Figure 4 of the main text, was constructed to simulate the cash flow utilization, and a system of differential equations was constructed on the basis of the model. The rate of money utilization depends on the number of children in the system each day, and the daily expenditure per child, as well as the rate of the flow of children out of the system. The total expenditure can be estimated by integration of the resulting differential equations.

Define:

t	number of days after the start of foster-care utilization study
k	child flow rate constant. A larger k denotes a faster discharge of foster children from the program; k is determined by examining prior agency data
C	represents a constant of integration, the value of which can be determined by substituting in any condition where all values for the variables can be determined.
$[\$]_t$	represents the amount of money spent after t days
$[\$]_o$	represents the initial monetary allocation at (t = 0)...i.e., on the first day
pdr	per diem rate

$$\frac{d[\$]}{dt} = [pdr][kids] \tag{10}$$

$$d = [\$] = [pdr][kids]dt = [pdr][kids]_o e^{-kt} dt \tag{11}$$

$$\int d[\$] = [pdr][kids]_o \int e^{-kt} dt \tag{12}$$

$$[\$] = [pdr][kids]_o \left(-\frac{1}{k}\right) e^{-kt} = C \tag{13}$$

On the first day, when kids have not yet been discharged:

$$[\$]_o = [pdr][kids]_o \left(-\frac{1}{k}\right) e^{-k(0)} + C \tag{14}$$

$$[\$]_o = [pdr][kids]_o \left(-\frac{1}{k}\right)(1) + C \tag{15}$$

$$C = [\$]_o + [pdr][kids]_o \left(-\frac{1}{k}\right) \tag{16}$$

Therefore, at time t:

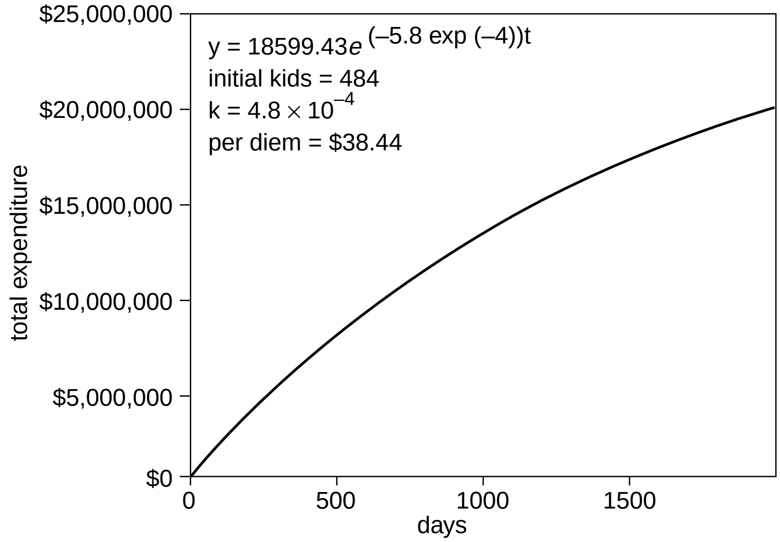
$$[\$]_t + [pdr][kids]_o \left(\frac{1}{k}\right) \tag{17}$$

$$[\$]_t = [\$]_o + [pdr][kids]_o \left(\frac{1}{k}\right) - [pdr][kids]_o \left(\frac{1}{k}\right) e^{-k(t)} \tag{18}$$

A plot of the solution to the differential equation representing expenditure utilization as a function of time is depicted in Figure 6 below.

FIGURE 6

Monetary Expenditure Over Time



Graph of monetary expenditure over time, calculated from the above expression. The curve fit is obtained from the 2nd-order polynomial curve-fitting routine in Cricket-Graph.

An equation can be derived that predicts the number of children that must be discharged in order to meet a certain monetary goal. By rearranging algebraically:

$$[\$]_t = [\$]_o + [pdr][kids]_o\left(\frac{1}{k}\right) - [pdr][kids]_o\left(\frac{1}{k}\right)e^{-k(t)} \tag{19}$$

$$[\$]_t - [\$]_o = [pdr][kids]_o\left(\frac{1}{k}\right) - [pdr][kids]_o\left(\frac{1}{k}\right)e^{-k(t)} \tag{20}$$

Recall:

$$[kids]_t = [kids]_o\, e^{-kt} \tag{21}$$

Substituting in:

$$[\$]_t = [\$]_o + [pdr][kids]_o\left(\frac{1}{k}\right) - [pdr]\left(\frac{1}{k}\right)[kids]_t^{-k(t)} \tag{22}$$

$$[pdr]\left(\frac{1}{k}\right)[kids]_t = [\$]_t - [\$]_o - [pdr][kids]_o\left(\frac{1}{k}\right) \tag{23}$$

$$[kids]_t = \frac{[\$]_t - [\$]_o\,[pdr][kids]_o\left(\frac{1}{k}\right)}{[pdr]\left(\frac{1}{k}\right)} \tag{24}$$

$$[kids]_t = k\left(\frac{[\$]_t - [\$]_o}{[pdr]}\right) - [kids]_o \tag{25}$$

Substituting the projected total expenditure and the projected number of kids:

$$[\text{kids}]_{\text{proj}} = k\left(\frac{[\$]_{\text{proj}} - [\$]_o}{[\text{pdr}]}\right) - [\text{kids}]_0 \tag{26}$$

$$[\text{kids}]_{\text{proj}} = \frac{k}{[\text{pdr}]}\left([\$]_{\text{proj}} - [\$]_o\right) - [\text{kids}]_0 \tag{27}$$

$$\Delta[\text{kids}] = [\text{kids}]_{\text{act}} - [\text{kids}]_{\text{proj}} \tag{28}$$

$$\Delta[\text{kids}] = [\text{kids}]_{\text{act}} - \left(\frac{k}{[\text{pdr}]}\left([\$]_{\text{proj}} - [\$]_o\right) - [\text{kids}]_0\right) \tag{29}$$

(Again it must be noted that boundary conditions exist on equation 21, that there can never be the condition where $[\text{kids}]_t - [\text{kids}]_o < 0$. Therefore, because equation 25 is derived from equation 21 the same restrictions must apply.)

A comparison between $[\text{kids}]_t$ and the number of children present in foster care at the time of the calculation provides one indicator of the effectiveness of an agency's implementation of the managed care objectives described in this paper.

About the Author

Fred Wulczyn is a Research Fellow at Chapin Hall. He is Co-Director of the Multistate Foster Care Data Archive. The Archive is a database, established by Chapin Hall, which includes foster care career histories for all children placed in state-supported substitute care in seven states. Currently, Mr. Wulczyn also serves as a member of the New York City Comptroller's Task Force on Foster Care Outcomes and Performance Standards. His most recent published articles include works that describe a statistical and methodological framework for analyzing the foster care experiences of children and the relationship between child welfare reform, community-based services, and managed care. Mr. Wulczyn has recently completed a four-site case study of managed care in child welfare.

Mr. Wulczyn has a wide range of experience in government and academia. He was Director of the Managed Care Initiative in Child Welfare for the New York State Department of Social Services. In that capacity, he developed and led the implementation of the HomeRebuilders Project in New York City, the nation's first managed care experiment with foster boarding home programs. Mr. Wulczyn has also been an Assistant Professor at the Columbia University School of Social Work. He holds an M.S.W. from the Graduate School of Social Work at Marywood University and a Ph.D. from the School of Social Service Administration at the University of Chicago.

Eileen Sheu, Ph.D., is a Research Associate at Chapin Hall. Her research interests primarily center around issues of administrative data and system dynamics, where the tools of mathematics are applied to obtain insights into large systems, such as foster care, involving movement or rate of change. However, in order to maintain the perspective that children cannot be reduced to numbers, Ms. Sheu can often be found donating her free time to the METRO Achievement Center, a tutoring and mentoring program for inner-city girls in Chicago.

Ms. Sheu received a B.S. from MIT and a Ph.D. in Inorganic Chemistry from the University of Notre Dame.

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